Cooperation and the theory of games

Milind Watve

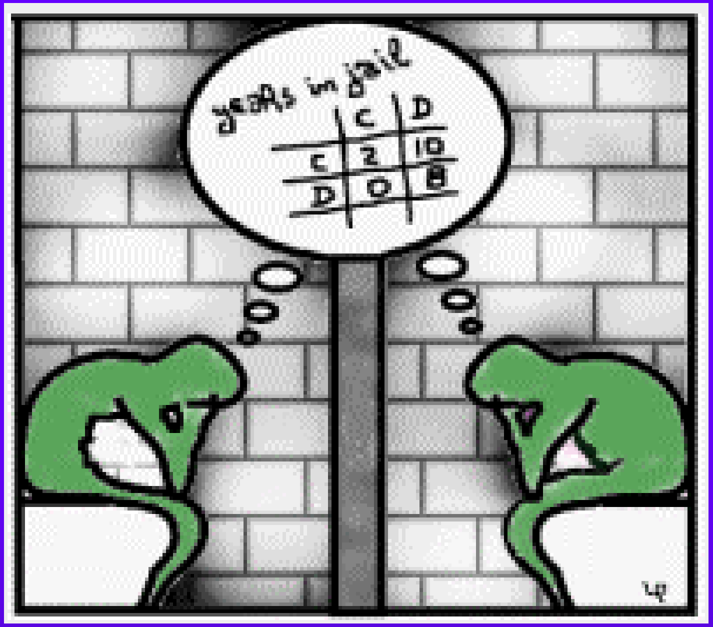
Department of Microbiology,

Abasaheb Garware College,

Karve Road, Pune

Game theory

- Optimization isn't one way
- When the success of your strategy is decided by the opponent's strategy
- Foundation by John von Newman "*The theory of games and economic behavior*".
- Shaping the game: John Nash, John Harsanyi, Reinhard Selten, Robert Aumann and Thomas Schelling.



John Maynard Smith: *Evolution and the theory of games*

ESS (evolutionary stable strategy): Players need not be rational.

If the payoff of an invader is less than the average of the standing population the strategy at home is ESS.

Prisoner's dilemma

Prisoner 2

Prisoner 1

	Cooperate (Do not confess)	**Defect (Confess)**
Cooperate (Do not confess)	Both get a 6 months jail sentence	Prisoner 2 is freed; Prisoner 1 gets 10 years in jail
Defect (Confess)	Prisoner 1 is freed; Prisoner 2 gets 10 years in jail	Both get a 3 year jail term

Both prisoners can cooperate with each other and get out of the jail in 6 months; but the greed of getting freed and the fear of getting 10 year jail term leads both the prisoners to defect with each other by confessing and get 3 years of jail term.

Prisoner's dilemma: a simpler approach

Player 2

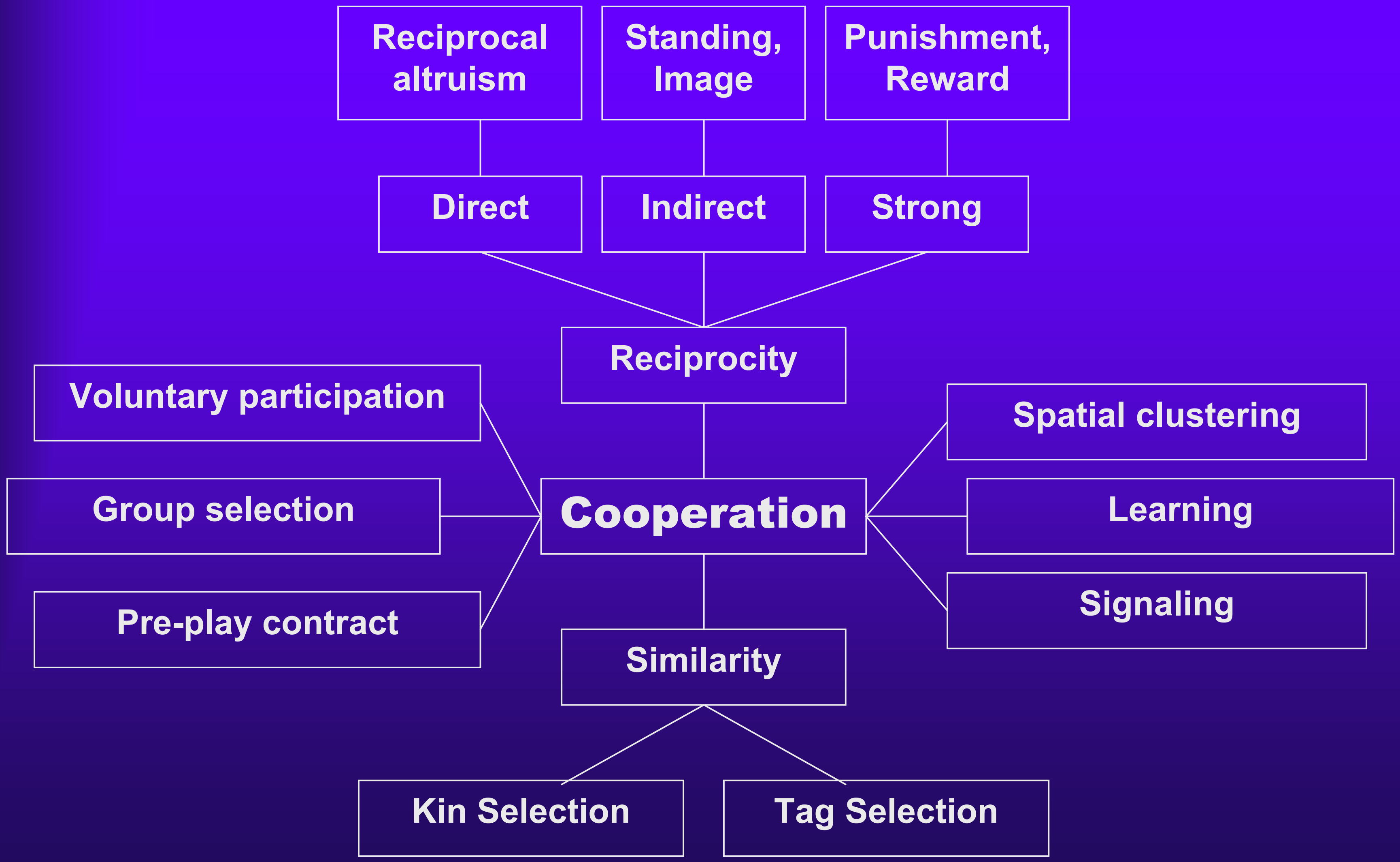
Player 1		Cooperate	Defect
	Cooperate	3, 3	0, 5
	Defect	5, 0	1, 1

Defecting is the best strategy no matter what the next player will do. But if both players defect, both get only 1 rather than 3 that they would got if both had cooperated with each other.

Variants of Prisoner's Dilemma game

- **Prisoner's Dilemma (PD)**
  - **Two – person PD**
    - **Simultaneous**
      - Both players play the moves simultaneously
    - **Alternating**
      - Players play the moves in alternate order
    - **Continuous**
      - Game is played continuously by increasing investments
  - **n – person PD**
    - **Tragedy of the commons**
      - Individuals over exploit a common resource
    - **Public goods game**
      - Everybody contributes for a public good and share the benefit equally

Strategies that evolve cooperation



Axelrod’s tournaments

Tit-for-Tat: The first grand slam winner

(i)“nice” – TFT is never the first to defect,

(ii)“retaliating” – TFT immediately defects to a defecting partner,

(iii)“forgiving” – resumes cooperation if the other player cooperates.

Tit-for-tat captures essence of reciprocal altruism

However, TFT works only if the probability of meeting the same opponent is high.

Also, it cannot work when individuals do mistakes.

TFT	C	C	C	D′	C	D	C	D	C	D…
TFT	C	C	C	C	D	C	D	C	D	C…

Punish but don’t tit

TFT player retaliates by defecting with a defecting partner and thus makes his own hands dirty.

PBDT, a strategy that has attractive features of generosity and forgiveness but retaliates by punishing rather than defection, can evolve cooperation even if probability of meeting the same opponent is very less. Also, it is highly robust against mistakes.

PBDT	**C**	**C**	**C**	**D′**	**C**	**C**	**C**	**C**	**C**	**C**…
PBDT	**C**	**C**	**C**	**C**	**P**	**C**	**C**	**C**	**C**	**C**…
TFT	**C**	**C**	**C**	**D′**	**C**	**C**	**C**	**C**	**C**	**C**…
PBDT	**C**	**C**	**C**	**C**	**P**	**C**	**C**	**C**	**C**	**C**…

C = cooperation

D = defection

P = cooperation with punishment

Evolutionary stability of Punish but don't tit

	C	*D*	*PBDT*
C	$\frac{b-c}{1-\omega},\frac{b-c}{1-\omega}$	$\frac{-c}{1-\omega},\frac{b}{1-\omega}$	$\frac{b-c}{1-\omega},\frac{b-c}{1-\omega}$
D	$\frac{b}{1-\omega},\frac{-c}{1-\omega}$	0,0	$b+\frac{\omega(b-y)}{1-\omega},-c+\frac{\omega(-c-x)}{1-\omega}$
PBDT	$\frac{b-c}{1-\omega},\frac{b-c}{1-\omega}$	$-c+\frac{\omega(-c-x)}{1-\omega},b+\frac{\omega(b-y)}{1-w}$	$\frac{b-c}{1-\omega},\frac{b-c}{1-\omega}$

ω is the probability that the same two players will meet again
b is the benefit of a cooperative act
c is the cost of cooperation
x is the cost of punishing
y is the penalty paid by a punished defector
We always assume $b > c > 0$; $y > x > 0$ and $b - y > - c - x$.

Then, PBDT will be evolutionary stable against defectors if,

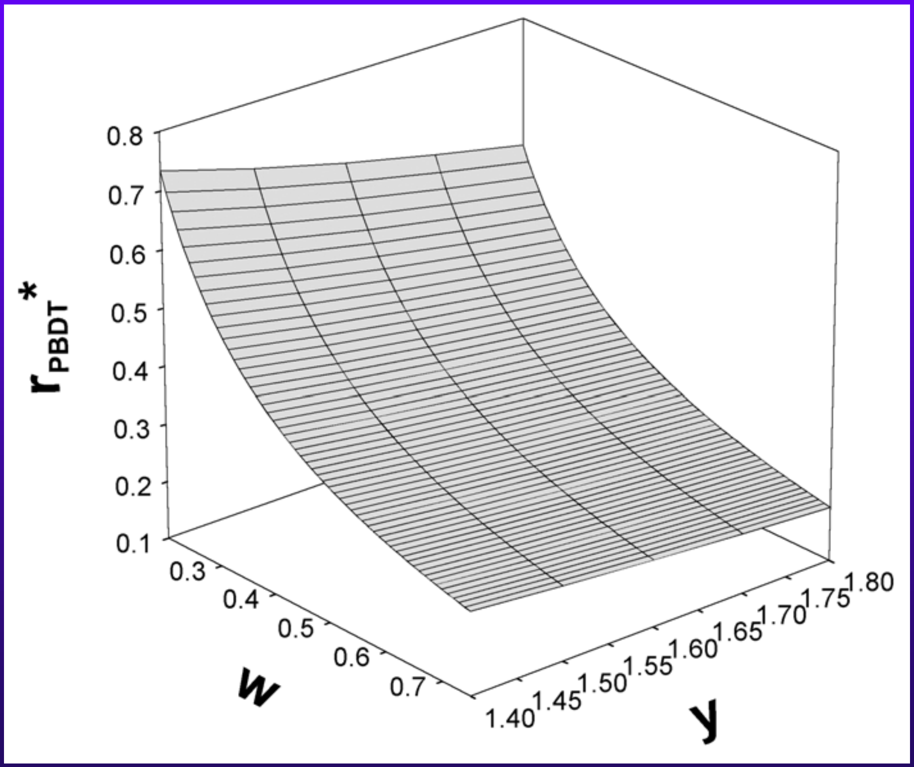
$$\omega \geq \frac{c}{y}$$

This is lesser than the ω required for evolutionary stability of TFT, which is $\geq \frac{c}{b}$ **, as b < y.**

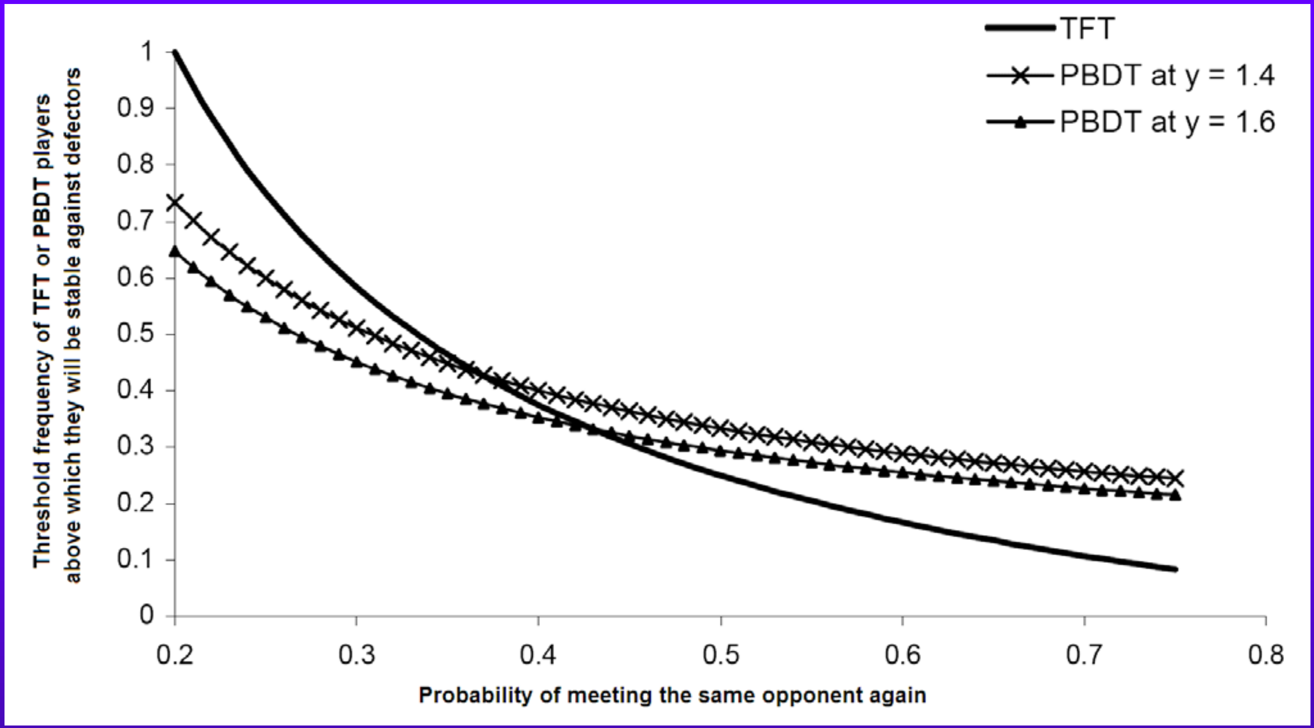
The minimum frequency of PBDT players necessary to evolve cooperation will be

$$r^* = \frac{c + \omega x}{(y + x)\omega}$$

Effect of ω and y on the minimum PBDT players required to stabilize cooperation



Cooperation in Prisoner's Dilemma: TFT versus PBDT



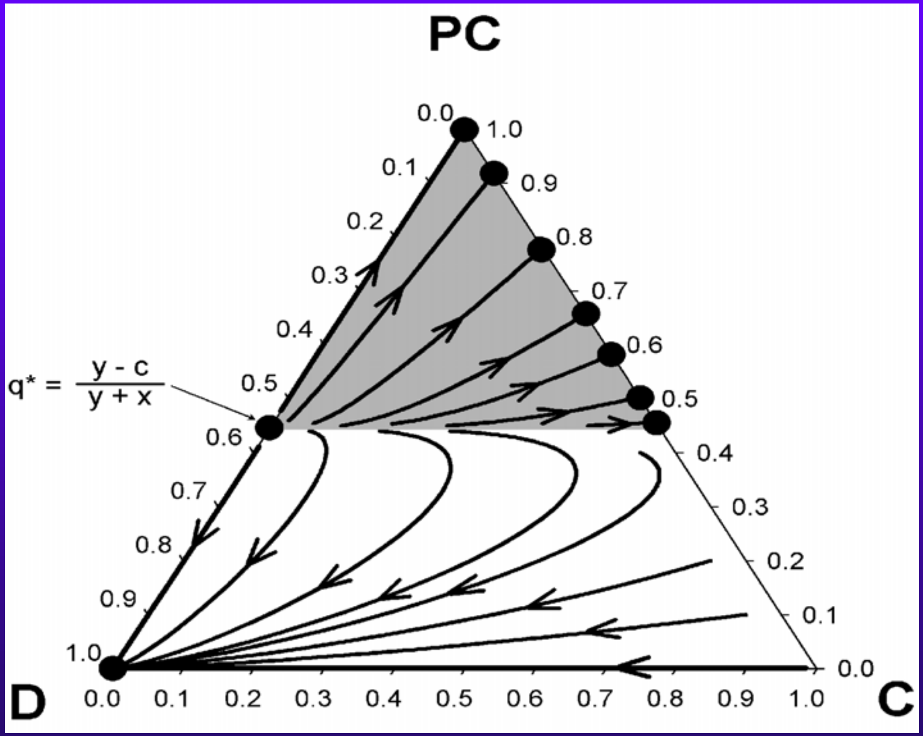
Both TFT and PBDT will perform equally if $$\omega^* = \frac{c(c+y+x-b)}{bx+cy}$$

If $\omega < \omega^*$, then PBDT works better than TFT. While, if $\omega > \omega^*$, TFT is better than PBDT.

Thus, PBDT is better than TFT in a large and less viscous population, however, in a small and viscous population, TFT is better than PBDT.

The problem with punishment

If punishment is costly for the people who engage in the punishing act, then non-punishing cooperators can exploit the punishing cooperators and act as the second-order free riders.



Then how will punishment evolve and get stable?

Discriminating defectors can stabilize punishment in a Prisoner's Dilemma

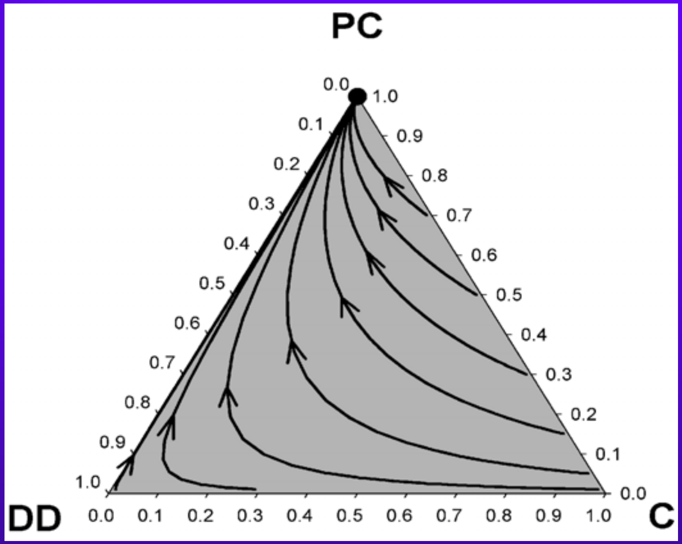
If punishers gain reputation, discriminating defectors will be able to discriminate between punishers and other players.

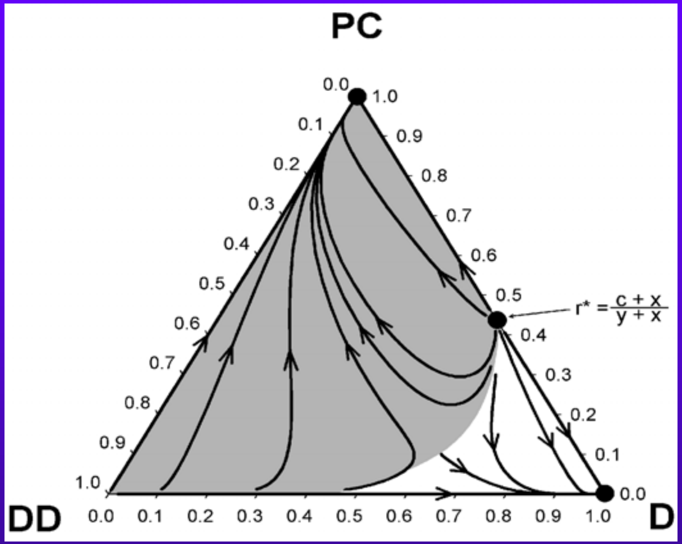
Such discriminating defectors will defect with other defectors and non punishing cooperators but they will cooperate with a punishing cooperator to avoid getting punished.

Discriminating defectors quickly replace simple defectors in presence of punishers since the former avoid punishment.

In the presence of discriminating defectors, non-punishing cooperators cannot free ride since they get the sucker's payoff with the discriminating defectors.

Discriminating defectors and the stability of punishment

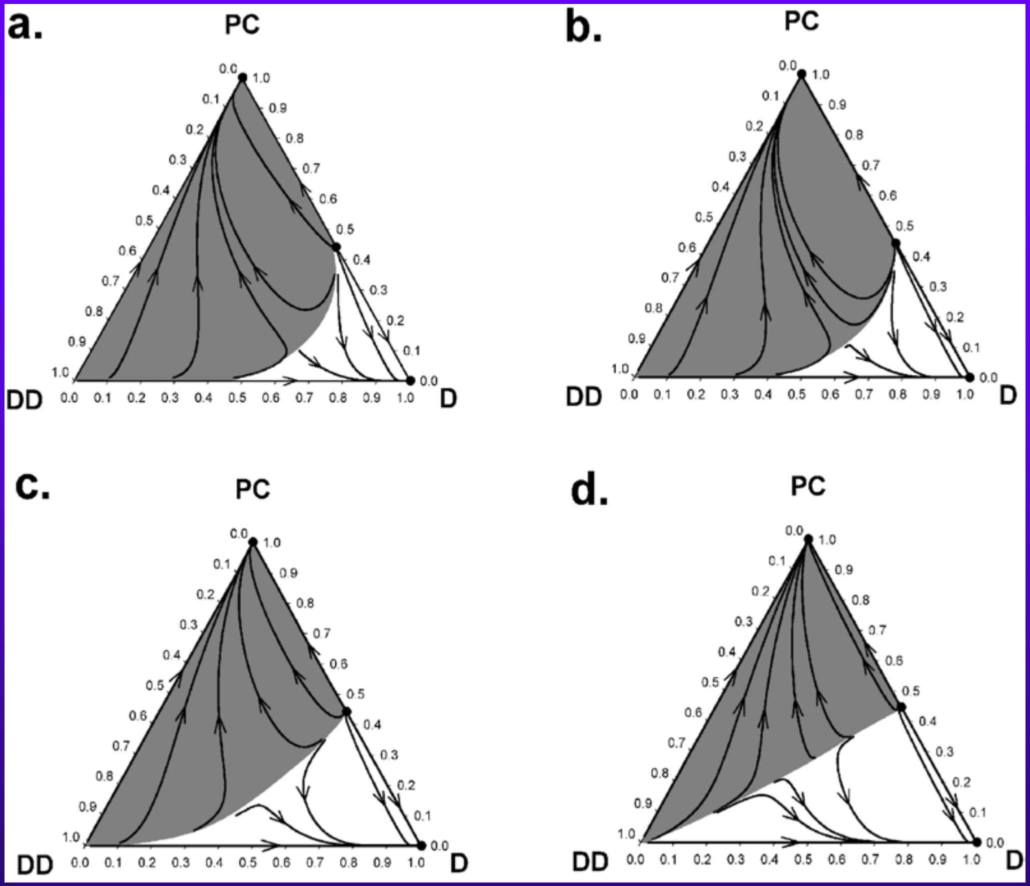




Cost of discrimination negligible

Cost of discrimination significant

Mistakes in discrimination



a. 1 % mistakes
b. 10% mistakes
c. 40% mistakes
d. 60% mistakes

Cooperating when strategies are hidden

Without a break