

Finding square-root using Newton-Raphson's method

B. W. Gore

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This note shows how to find square-root using Newton-Raphson's method. Let R be the required number and let x be its square-root. Then,

$$x = \sqrt{R} \Rightarrow x^2 = R \Rightarrow x^2 - R = 0$$

$$\therefore f(x) \equiv x^2 - R = 0 \Rightarrow f'(x) = 2x$$

From Newton-Raphson's method we have:

$$\begin{aligned} x_{i+1} &= x_i - \frac{f(x_i)}{f'(x_i)} \\ &= x_i - \frac{x_i^2 - R}{2x_i} \\ &= \frac{2x_i^2 - x_i^2 + R}{2x_i} \\ &= \frac{1}{2} \left[x_i + \frac{R}{x_i} \right] \end{aligned}$$

We can start the iteration with $X_0 = R/2$ and proceed. A sample output of the iterations looks like this:

Enter the number to find sqrt: 210.25

0	105.125000
1	53.562500

2	28.743910
3	18.029252
4	14.845428
5	14.504019
6	14.500001
7	14.500000

$\text{sqrt}(210.250000) = 14.500000$